Weyl Representation of Dirac Algebra

One common representation of the Dirac algebra is called the *Weyl* or *chiral* representation. It is given in 2×2 block form as follows.

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

Explicitly in 4×4 form it is.

$$\gamma^0 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

In this representation $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ is given in 2×2 form by

$$\gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

and in 4×4 form it is.

$$\gamma^5 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

It is useful to note the symmetry of each Dirac matrix in this representation.

$$\gamma^{0T} = \gamma^0 \qquad \gamma^{1T} = -\gamma^1 \qquad \gamma^{2T} = \gamma^2 \qquad \gamma^{3T} = -\gamma^3$$

We can write this compactly as follows.

$$\gamma^{iT} = (-1)^i \gamma^i$$