Scattering

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The difference of two forward, time-like vectors with the same invariant magnitudes is (1) space-like or (2) identically zero. This situation occurs in elastic scattering with single photon (or other vector particle) exchange. Consider two vectors, k and k', with the same mass m and positive energy, i.e. $k^2 = m^2$, $k'^2 = m^2$, $k_0 > 0$, and $k'_0 > 0$. Under these circumstances, we can boost to the rest frame for k'. In this frame $k' = (m, 0, 0, 0)$ and $k = (E, \vec{p})$ where $E^2 = \vec{p}^2 + m^2$. The square of the difference is $q^2 = k^2 + k'^2 - 2k \cdot k' = 2m^2 - 2Em = -2m(E - m) \leq 0$. The condition for $q^2 = 0$ is $E = m$ or $\vec{p} = 0$ which implies that $k = k'$. Therefore, $q^2 < 0$ or $q = 0$; both of which satisfy $q^2 \leq 0$.

The sum of two forward, time-like vectors with the same invariant magnitudes is (1) time-like or (2) light-like if the two vectors are identical and have a vanishing magnitude. This situation occurs in pair annihilation to a photon (or other vector particle). Again consider two vectors, k and k', with the same mass m and positive energy, i.e. $k^2 = m^2$, $k'^2 = m^2$, $k_0 > 0$, and $k'_0 > 0$. Under these circumstances, we can boost to the center of mass frame. In this frame $k = (E, \vec{p})$ and $k' = (E, -\vec{p})$ where $E^2 = \vec{p}^2 + m^2$. The sum is $q = k + k' = (2E, 0, 0, 0)$ with $q^2 = 4E^2 = 4m^2 + 4\vec{p}^2$. The condition for $q^2 = 0$ is $m = 0$ and $\vec{p} = 0$ which implies that $k = k'$. Therefore, $q^2 > 0$ or $k = k'$ and $m = 0$; both of which satisfy $q^2 \geq 4m^2 \geq 0$.