Quantum Fields

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The Schrodinger fields are functions of space only, denoted $O_S(\vec{x})$, that satisfy the following.

$$e^{-i\vec{a}\cdot\vec{P}} O_S(x) e^{i\vec{a}\cdot\vec{P}} = O_S(\vec{x} + \vec{a})$$

The Minkowski Hamiltonian fields are defined by the following.

$$O_M(t, \vec{x}) = e^{itH} O_S(\vec{x}) e^{-itH}$$

These fields are usually denoted as O_H, but to distinguish them from their Euclidean counterparts we denoted them with M instead of H. Also note that most operators in books are Hamiltonian and usually the H is dropped altogether. The Minkowski fields satisfy the following.

$$e^{ia\cdot P} O_M(x) e^{-ia\cdot P} = O_M(x + a)$$

We derive the above relationship as follows.

$$\begin{aligned}
&e^{ia\cdot P} O_M(x) e^{-ia\cdot P} = \\
&e^{ia_0 H} e^{-i\vec{a}\cdot\vec{P}} e^{ix_0 H} O_S(\vec{x}) e^{-ix_0 H} e^{-ia_0 H} e^{i\vec{a}\cdot\vec{P}} = \\
&e^{i(x_0+a_0)H} e^{-i\vec{a}\cdot\vec{P}} O_S(\vec{x}) e^{i\vec{a}\cdot\vec{P}} e^{-i(x_0+a_0)H} = \\
&e^{i(x_0+a_0)H} O_S(\vec{x} + \vec{a}) e^{-i(x_0+a_0)H} = \\
&O_M(x + a)
\end{aligned}$$

The Euclidean Hamiltonian fields are defined by the following.

$$O_E(t, \vec{x}) = e^{tH} O_S(\vec{x}) e^{-tH}$$

The Euclidean fields satisfy the following.

$$e^{a_0 H} e^{-i\vec{a}\cdot\vec{P}} O_E(x) e^{-a_0 H} e^{i\vec{a}\cdot\vec{P}} = O_E(x+a)$$

We derive the above relationship as follows.

$$\begin{aligned}
& e^{a_0 H} e^{-i\vec{a}\cdot\vec{P}} O_E(x) e^{-a_0 H} e^{i\vec{a}\cdot\vec{P}} = \\
& e^{a_0 H} e^{-i\vec{a}\cdot\vec{P}} e^{x_0 H} O_S(\vec{x}) e^{-x_0 H} e^{-a_0 H} e^{i\vec{a}\cdot\vec{P}} = \\
& e^{(x_0+a_0)H} e^{-i\vec{a}\cdot\vec{P}} O_S(\vec{x}) e^{i\vec{a}\cdot\vec{P}} e^{-(x_0+a_0)H} = \\
& e^{(x_0+a_0)H} O_S(\vec{x}+\vec{a}) e^{-(x_0+a_0)H} = \\
& O_E(x+a)
\end{aligned}$$

The definitions of Minkowski and Euclidean fields are identical at $t = 0$.

$$O_E(0,\vec{x}) = O_M(0,\vec{x})$$

This is a very important result. It tells us how to relate matrix elements of Euclidean operators (which can be calculated on the lattice) to the same matrix elements of the corresponding Minkowski operator by matching the operators at $t = 0$.